Math 2050, HW 5

Q1. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous function and $S = \{x \in \mathbb{R} : f(x) = 0\}$. Show that S is closed in the sense that if $x_n \in S$ and $x_n \to x$, then $x \in S$.

Q2. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous functions such that

$$f(m2^{-n}) = m2^{-n}$$

for all $m \in \mathbb{Z}, n \in \mathbb{N}$. Show that $f(x) = x$ for all $x \in \mathbb{R}$.

Q3. Using the ε, δ terminology to show that

(a)

$$\lim_{x \to 2} \sqrt{\frac{2x+1}{x+3}} = 1.$$

(b)

$$\lim_{x \to 1} \frac{x^2 - 3x}{x+3} = \frac{-1}{2}.$$

Q4. Show that the following limit does not exist

$$\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$$

Q5. If $f : A \to \mathbb{R}_{\geq 0}$ and c is a cluster point of A so that f has a limit $L \geq 0$ at c. Show that $\sqrt{f}$ has limit $\sqrt{L}$ at c.